1. Find the real roots of $x^4 + 4x^3 + 6x^2 + 4x - 15$.

   Answer: 1, −3

   Solution: Note that $x^4+4x^3+6x^2+4x-15 = (x+1)^4-2^4 = ((x+1)^2+4)((x+1)^2-4)$. Thus, the roots of $x^4+4x^3+6x^2+4x-15$ are the roots of $(x+1)^2-4$, so the roots are $x = \boxed{1, -3}$.

2. Compute

$$\sum_{m=1}^{2015} \sum_{k=m-2015}^{m-2} \frac{1}{m^2+k^2-2mk-m+k}$$

   Answer: 2014

   Solution: Note that the denominator of the fraction can be factored. Then, if we make the substitution $n = m-k$, then the entire inner sum can be written without m, and becomes a simple telescoping sum. The answer follows.

$$\begin{aligned}
\sum_{m=1}^{2015} \sum_{k=m-2015}^{m-2} \frac{1}{m^2+k^2-2mk-m+k} &= \sum_{m=1}^{2015} \sum_{k=m-2015}^{m-2} \frac{1}{(m-k)(m-k-1)} \\
&= \sum_{m=1}^{2015} \sum_{n=2}^{2015} \frac{1}{n(n-1)} \\
&= 2015 \sum_{n=2}^{2015} \frac{1}{n(n-1)} \\
&= 2015 \cdot \frac{2014}{2015} \\
&= \boxed{2014}
\end{aligned}$$

3. Find the minimal value of

$$\left(\frac{x+y}{x-y}\right)^2 + \left(\frac{y+z}{y-z}\right)^2 + \left(\frac{z+x}{z-x}\right)^2$$

   where $x \neq y \neq z$ are real numbers.

   Answer: 2

   Solution: Set $a = \frac{x+y}{x-y}$, $b = \frac{y+z}{y-z}$, $c = \frac{z+x}{z-x}$. Observe that

$$(a+1)(b+1)(c+1) = \frac{2x \cdot 2y \cdot 2z}{(x-y)(y-z)(z-x)} = \frac{2y \cdot 2z \cdot 2x}{(x-y)(y-z)(z-x)} = (a-1)(b-1)(c-1).$$

   This means that $ab+bc+ca = -1$, so therefore $a^2+b^2+c^2 = (a+b+c)^2+2$ and hence $a^2+b^2+c^2 \geq \boxed{2}$.

   Finally, we show that this is achievable: take $x=-1$, $y=0$, $z=1$.