

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Given that x and y are nonnegative integers, compute the number of pairs (x, y) such that $5x + y = 20$.
2. $f(x) = x^2 + bx + c$ is a function with the property that the x -coordinate of the vertex is equal to the positive difference of the two roots of $f(x)$. Given that $c = 48$, compute b .
3. Suppose we have a function $f(x)$ such that $f(x)^2 = f(x - 5)f(x + 5)$ for all integers x . Given that $f(1) = 1$ and $f(16) = 8$, what is $f(2016)$?
4. Suppose that we have the following set of equations

$$\log_2 x + \log_3 x + \log_4 x = 20$$

$$\log_4 y + \log_9 y + \log_{16} y = 16$$

Compute $\log_x y$.

5. Let $\{a_n\}$ be the arithmetic sequence defined as $a_n = 2(n - 1) + 6$ for all $n \geq 1$. Compute

$$\sum_{i=1}^{\infty} \frac{1}{a_i a_{i+2}}$$

6. Let a, b, c, d, e, f be non-negative real numbers. Suppose that $a + b + c + d + e + f = 1$ and $ad + be + cf \geq \frac{1}{18}$. Find the maximum value of $ab + bc + cd + de + ef + fa$.
7. Let f be a continuous real-valued function defined on the positive real numbers. Determine all f such that for all positive real x, y we have $f(xy) = xf(y) + yf(x)$ and $f(2016) = 1$.
8. Find the maximum of the following expression:

$$21 \cos \theta + 18 \sin \theta \sin \phi + 14 \sin \theta \cos \phi.$$

9. a, b, c, d satisfy the following system of equations

$$ab + c + d = 13$$

$$bc + d + a = 27$$

$$cd + a + b = 30$$

$$da + b + c = 17$$

Compute the value of $a + b + c + d$.

10. Define a sequence of numbers $a_{n+1} = \frac{(2+\sqrt{3})a_n+1}{(2+\sqrt{3})-a_n}$ for $n > 0$, and suppose that $a_1 = 2$. What is a_{2016} ?