1. Mark takes a two digit number x and forms another two digit number by reversing the digits of x. He then sums the two values, obtaining a value which is divisible by 13. Compute the smallest possible value of x.

   Answer: 49

   Solution: Let $x = AB$, where $1 \le A \le 9$ and $0 \le B \le 9$. We observe that

   $$AB + BA = 10(A+B) + (A+B) = 11(A+B)$$

   Hence, the resulting sum will be divisible by 11 and $A+B$, and since the sum must be divisible by 13, we have $A+B = 13$. x is minimized when we choose $A = 4$ and $B = 9$, hence $x = \boxed{49}$.

2. Let $p(x) = x^4 - 10x^3 + cx^2 - 10x + 1$, where c is a real number. Given that $p(x)$ has at least one real root, what is the maximum value of c?

   Answer: 27

   Solution: Note that the products of the roots of $p(x)$ are 1. For now, we guess that $p(x)$ has two pairs of roots, with each pair having a product of 1. Then we can write $p(x)$ as

   $$\begin{aligned} p(x) &= (x^2 - ax + 1)(x^2 - bx + 1) \\ &= x^4 - (a+b)x^3 + (1 + ab + 1)x^2 - (a+b)x + 1 \end{aligned}$$

   Hence, we must maximize ab subject to the constraint that $a + b = 10$. By AM-GM, ab is maximized when $a = b = 5$, and we have that $c = 1 + 25 + 1 = 27$.

   To show that no larger value of c exists, we can factorize $p(x)$ when $c = 27$:

   $$\begin{aligned} p(x) &= x^4 - 10x^3 + 27x^2 - 10x + 1 \\ &= (x^2 - 5x + 1)(x^2 - 5x + 1) \ge 0 \end{aligned}$$

   Since $x^2 > 0$, increasing our choice of c to some c' would result in $p(x) + (c' - c)x^2 > 0$ for all x, implying that $p(x)$ has no real roots.

   We thus conclude that our answer is $c = \boxed{27}$.

3. x satisfies the equation $(1+i)x^3 + 8ix^2 + (-8+8i)x + 36 = 0$. Compute the largest possible value of $|x|$.

   Answer: $\mathbf{3\sqrt{2}}$

   Solution: Recall that a complex number can be represented in terms of a direction and a magnitude, and observe the direction of the complex coefficients. From the first three coefficients, we see a factor of $1+i$ in the x^3 coefficient, a factor of i in the x^2 coefficient, and a factor of $-1+i$ in the x coefficient. Thus, making a substitution may simply the equation.

   Let $x = (1+i)y$. First, we observe that $(1+i)^2 = 2i$; $(1+i)^3 = -2+2i$; and $(1+i)^4 = -4$. Making the substitution, we have

   $$\begin{aligned} 0 &= (1+i)x^3 + 8ix^2 + (-8+8i)x + 36 \\ 0 &= (1+i)((1+i)y)^3 + 8i((1+i)y)^2 + (-8+8i)((1+i)y) + 36 \\ 0 &= (1+i)(1+i)^3y^3 + 4(2i)(1+i)^2y^2 + 4(-2+2i)(1+i)y + 36 \\ 0 &= -4y^3 - 4 \cdot 4y^2 - 4 \cdot 4y + 36 \\ 0 &= y^3 + 4y^2 + 4y - 9 \end{aligned}$$

Since the coefficients sum to equal 1, $y = 1$ is a solution. Finding the remaining solutions for y and solving for x, we have

$$
\begin{aligned}
0 &= (y-1)(y^2+5y+9) \\
y &= 1, \frac{-5+i\sqrt{11}}{2}, \frac{-5-i\sqrt{11}}{2} \\
x &= 1+i, (1+i)\frac{-5+i\sqrt{11}}{2}, (1+i)\frac{-5-i\sqrt{11}}{2}
\end{aligned}
$$

Clearly, $|x|$ is maximized when we choose x to be either the second or third root. Choosing the second root, we compute $|x|$ to be

$$|x| = \left|(1+i)\frac{-5+i\sqrt{11}}{2}\right| = \sqrt{1+1}\sqrt{\frac{25+11}{4}} = \sqrt{2}\sqrt{9} = \boxed{3\sqrt{2}}$$