

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Evaluate

$$\int_{-2}^2 (x^3 + 2x + 1) \, dx.$$

2. Find

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x + x^3) - x}{x^2}.$$

3. Find the largest possible value for the slope of a tangent line to the curve $f(x) = \frac{1}{3+x^2}$.

4. An empty, inverted circular cone has a radius of 5 meters and a height of 20 meters. At time $t = 0$ seconds, the cone is empty, and at time $t \geq 0$ we fill the cone with water at a rate of $4t^2$ cubic meters per second. Compute the rate of change of the height of water with respect to time, at the point when the water reaches a height of 10 meters.

5. Compute

$$\int_0^{\frac{\pi}{2}} \sin(2016x) \cos(2015x) \, dx.$$

6. Let $f(x)$ be a function defined for $x > 1$ such that $f''(x) = \frac{x}{\sqrt{x^2-1}}$ and $f'(2) = \sqrt{3}$. Compute the length of the graph of $f(x)$ on the domain $x \in (1, 2]$.

7. Let the function $f : [1, \infty) \rightarrow \mathbb{R}$ be defined as $f(x) = x^{2\ln(x)}$. Compute

$$\int_1^{\sqrt{e}} (f(x) + f^{-1}(x)) \, dx.$$

8. Calculate $f(3)$, given that $f(x) = x^3 + f'(-1)x^2 + f''(1)x + f'(-1)f(-1)$.

9. Compute

$$\int_1^e \frac{\ln(x)}{(1 + \ln(x))^2} \, dx.$$

10. For $x \geq 0$, let $\mathcal{R}$ be the region in the plane bounded by the graphs of the line $\ell : y = 4x$ and $y = x^3$. Let V be the volume of the solid formed by revolving $\mathcal{R}$ about line ℓ . Then V can be expressed in the form $\frac{\pi \cdot 2^a}{b\sqrt{c}}$, where a, b , and c are positive integers, b is odd, and c is not divisible by the square of a prime. Compute $a + b + c$.