Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Alice can bake a pie in 5 minutes. Bob can bake a pie in 6 minutes. Compute how many more pies Alice can bake than Bob in 60 minutes.

2. Ben likes long bike rides. On one ride, he goes biking for six hours. For the first hour, he bikes at a speed of 15 miles per hour. For the next two hours, he bikes at a speed of 12 miles per hour. He remembers biking 90 miles over the six hours. Compute the average speed, in miles per hour, Ben biked during the last three hours of his trip.

3. Compute the perimeter of a square with area 36.

4. Two ants are standing side-by-side. One ant, which is 4 inches tall, casts a shadow that is 10 inches long. The other ant is 6 inches tall. Compute, in inches, the length of the shadow that the taller ant casts.

5. Compute the number of distinct line segments that can be drawn inside a square such that the endpoints of the segment are on the square and the segment divides the square into two congruent triangles.

6. Emily has a cylindrical water bottle that can hold 1000π cubic centimeters of water. Right now, the bottle is holding 100π cubic centimeters of water, and the height of the water is 1 centimeter. Compute the radius of the water bottle.

7. Given that x and y are nonnegative integers, compute the number of pairs (x, y) such that $5x + y = 20$.

8. A sequence a_n is recursively defined where $a_n = 3(a_{n-1} - 1000)$ for $n > 0$. Compute the smallest integer x such that when $a_0 = x$, $a_n > a_0$ for all $n > 0$.

9. Compute the probability that two random integers, independently chosen and both taking on an integer value between 1 and 10 with equal probability, have a prime product.

10. If x and y are nonnegative integers, both less than or equal to 2, then we say that (x, y) is a friendly point. Compute the number of unordered triples of friendly points that form triangles with positive area.

11. Cindy is thinking of a number which is 4 less than the square of a positive integer. The number has the property that it has two 2-digit prime factors. What is the smallest possible value of Cindy's number?

12. Winona can purchase a pencil and two pens for 250 cents, or two pencils and three pens for 425 cents. If the cost of a pencil and the cost of a pen does not change, compute the cost in cents of five pencils and six pens.

13. Colin has an app on his phone that generates a random integer betwen 1 and 10. He generates 10 random numbers and computes the sum. Compute the number of distinct possible sums Colin can end up with.

14. A circle is inscribed in a unit square, and a diagonal of the square is drawn. Find the total length of the segments of the diagonal not contained within the circle.

15. A class of six students has to split into two indistinguishable teams of three people. Compute the number of distinct team arrangements that can result.

16. A unit square is subdivided into a grid composed of 9 squares each with sidelength $\frac{1}{3}$. A circle is drawn through the centers of the 4 squares in the outermost corners of the grid. Compute the area of this circle.

17. There exists exactly one positive value of k such that the line $y = kx$ intersects the parabola $y = x^2 + x + 4$ at exactly one point. Compute the intersection point.

18. Given an integer x, let $f(x)$ be the sum of the digits of x. Compute the number of positive integers less than 1000 where $f(x) = 2$.

19. Let $ABCD$ be a convex quadrilateral with $BA = BC$ and $DA = DC$. Let E and F be the midpoints of BC and CD respectively, and let BF and DE intersect at G. If the area of $CEGF$ is 50, what is the area of $ABGD$?

20. Compute all real solutions to $16^x + 4^{x+1} - 96 = 0$.

21. At an M&M factory, two types of M&Ms are produced, red and blue. The M&Ms are transported individually on a conveyor belt. Anna is watching the conveyor belt, and has determined that four out of every five red M&Ms are followed by a blue one, while one out of every six blue M&Ms is followed by a red one. What proportion of the M&Ms are red?

22. ABCDEFGH is an equiangular octagon with side lengths 2, $4\sqrt{2}$, 1, $3\sqrt{2}$, 2, $3\sqrt{2}$, 3, and $2\sqrt{2}$, in that order. Compute the area of the octagon.

23. The cubic $f(x) = x^3 + bx^2 + cx + d$ satisfies $f(1) = 3$, $f(2) = 6$, and $f(4) = 12$. Compute $f(3)$.

24. Given a unit square, two points are chosen uniformly at random within the square. Compute the probability that the line segment connecting those two points touches both diagonals of the square.

25. Compute the remainder when:

$$5\underbrace{6666\ldots6666}_{\text{2016 sixes}}5$$

is divided by 17.