

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. A circle is inscribed in a unit square, and a diagonal of the square is drawn. Find the total length of the segments of the diagonal not contained within the circle.
2. Points D and E are chosen on the exterior of $\triangle ABC$ such that $\angle ADC = \angle BEC = 90^\circ$. If $\angle ACB = 40^\circ$, $AD = 7$, $CD = 24$, $CE = 15$, and $BE = 20$, what is the measure of $\angle ABC$ in degrees?
3. Let $ABCD$ be a convex quadrilateral with $BA = BC$ and $DA = DC$. Let E and F be the midpoints of BC and CD respectively, and let BF and DE intersect at G . If the area of $CEGF$ is 50, what is the area of $ABGD$?
4. Let ABC be a right triangle with $\angle ABC$ right. Let D be a point on segment AC and let E be a point on ray AB past B such that $AC \perp DE$. Segment DE intersects segment BC at a point F such that $DF = FE$. Suppose that $\frac{CD}{BE} = 3$. Compute $\frac{AB}{AD}$.
5. Plane A passes through the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Plane B is parallel to plane A , but passes through the point $(1, 0, 1)$. Find the distance between planes A and B .
6. Let ABC be a triangle with $AB = 5$ and $AC = 4$. Let D be the reflection of C across AB , and let E be the reflection of B across AC . D and E have the special property that D, A, E are collinear. Finally, suppose that lines DB and EC intersect at a point F . Compute the area of $\triangle BCF$.
7. A circle intersects the y -axis at two points $(0, a)$ and $(0, b)$ and is tangent to the line $x + 100y = 100$ at $(100, 0)$. Compute the sum of all possible values of $ab - a - b$.
8. In rectangle $ABCD$, point E is chosen on AB and F is the foot of E onto side CD such that the circumcircle of $\triangle ABF$ intersects line segments AD and BC at points G and H respectively. Let S be the intersection of EF and GH , and T the intersection of lines EC and DS . If $\angle SFT = 15^\circ$, compute the measure of $\angle CSD$.
9. In quadrilateral $ABCD$, $AC = 5$, $CD = 7$, and $AD = 3$. The angle bisector of $\angle CAD$ intersects CD at E . If $\angle CBD = 60^\circ$ and $\angle AED = \angle BEC$, compute the value of $AE + BE$.
10. Circle ω_1 has diameter AB , and circle ω_2 has center A and intersects ω_1 at points C and D . Let E be the intersection of AB and CD . Point P is chosen on ω_2 such that $PC = 8$, $PD = 14$, and $PE = 7$. Find the length of PB .