Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Kevin plants corn and cotton. Once he harvests the crops, he has 30 pounds of corn and x pounds of cotton. Corn sells for \$5 per pound and cotton sells for \$10 per pound. If Kevin sells all his corn and cotton for a total of \$640, then compute x.

2. $ABCD$ is a square where $AB = \sqrt{2016}$. Let X be a point on AB and Y be a point on CD such that $AX = CY$. Compute the area of trapezoid $AXYD$.

3. If the integer n leaves a remainder of 4 when divided by 5, then what is the largest possible remainder when $2n$ is divided by 15?

4. Let $d(n)$ represent the sum of the digits of the integer n. For example,
$$d(2016) = 2 + 0 + 1 + 6 = 9.$$
For how many positive 3-digit integers k is it true that $d(k) > d(k + 1)$?

5. Let A, B, and C be three points on circle O such that AC is a diameter of O. Extend line AC to a point D such that DB is tangent to O at B, and suppose that $\angle ADB = 20^\circ$. Compute $\angle ACB$.

6. A group of n people, including Caroline and her best friend Catherine, stand in random order around a circle, with each order being equally likely. If the probability that Catherine is adjacent to Caroline is $\frac{1}{9}$, then what is the value of n?

7. The polynomial $P(x) = x^4 + 4x^3 + 8x^2 + 8x + 4$ is the square of another polynomial $Q(x) = ax^2 + bx + c$ with positive coefficients a, b, and c. Compute $4a + 2b + c$.

8. A physics class has 25 students, two of whom are Alex and Justin. Three students are chosen uniformly at random for a lab demonstration. What is the probability that at least one of Alex and Justin is chosen?

9. Natasha carries a paintbrush of length 1 and walks around the perimeter of a regular 2016-gon with side length 1, painting all of the area outside the polygon that she can reach. What is the area that Natasha paints?

10. Let S be the set of values which can be written as the sum of five consecutive perfect squares. What is the smallest element of S which is divisible by 17?

11. Consider 6 points, two of which are A and B, such that each pair of points is connected by a line segment and no three points are collinear. Compute the number of distinct paths from A to B such that no point is visited more than once.

12. Let $f(x) = 3x^2 + 2x + 6$. Two distinct lines l_1 and l_2 exist that are tangent to $f(x)$ and intersect at the origin. Given that l_1 is tangent to $f(x)$ at $(x_1, f(x_1))$ and that l_2 is tangent to $f(x)$ at $(x_2, f(x_2))$, compute x_1x_2.

13. One day, Connie plays a game with a fair 6-sided die. Connie rolls the die until she rolls a 6, at which point the game ends. If she rolls a 6 on her first turn, Connie wins 6 dollars. For each subsequent turn, Connie wins $\frac{1}{6}$ of the amount she would have won the previous turn. What is Connie's expected earnings from the game?

14. Calculate the positive root of the polynomial $(x-1)(x-2)(x-3)(x-4)-63$.

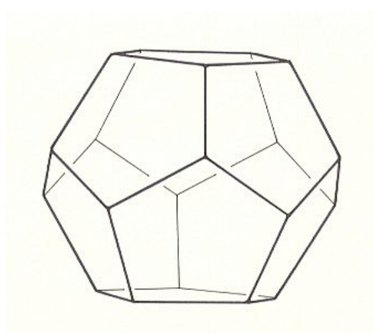
15. Consider an equilateral triangle $\triangle ABC$ with unit side length. Let M be the midpoint of side AB. A ball is released in a straight line from M and bounces off the side BC at a point D. Then, it bounces off the side CA at a point E and lands exactly at B. By the law of reflection, we have $\angle BDM = \angle CDE$ and $\angle CED = \angle AEB$. Calculate $MD + DE + EB$, the distance that the ball travels before reaching B.

16. Suppose that 12 points are marked evenly around a unit circle. How many pairs of distinct chords are there, with each chord having endpoints among the 12 points, such that the chords are parallel?

17. In the 3-D coordinate plane, two squares of side length 2, $ABCD$ and $EFGH$, are arranged with the sides of $ABCD$ lying parallel to the x- and y-axes and the diagonals of $EFGH$ lying along the y- and z-axes. In addition, the two squares share the same center point, A, B, G, C, D, E are coplanar, and FH is orthogonal to $ABCD$. Compute the volume of the solid $ABGHEFCD$.

18. Let F be the Fibonacci sequence of numbers, where $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. The sum $\sum_{i=1}^{2016} F_{3i}$ can be written in the form $\frac{1}{a}(F_b - c)$, where a, b, c are positive integers, $a > 1$, and b is minimal. Compute $a + b + c$.

19. A regular dodecahedron is a figure with 12 identical pentagons for each of its faces. Let x be the number of ways to color the faces of the dodecahedron with 12 different colors, where two colorings are identical if one can be rotated to obtain the other. Compute $\frac{x}{12!}$.



20. Compute
$$\int_0^3 \sqrt{\sqrt{x+1}-1}\,dx.$$

21. Let ω be a root of the polynomial $\omega^{2016} + \omega^{2015} + \cdots + \omega + 1$. Compute the sum
$$\sum_{i=0}^{2016} (1+\omega^i)^{2017}.$$

22. A lattice point is a pair (x, y) where x and y are integers. How many lattice points are on the graph of $y = \sqrt{x^2 + 60^4} + 2016$?

23. Compute

$$\sum_{\substack{p,q+1\geq 1\\ \gcd(p,q)=1}} \frac{1}{3^{p+q}-1}.$$

24. Let ABC be a right triangle with $\angle C = 90°$. The angle bisector of $\angle ACB$ intersects AB at D. In addition, the angle bisector of $\angle ADC$ intersects AC at E, and the angle bisector of $\angle BDC$ intersects BC at F. Suppose that $EC = 6$ and $FC = 8$, and let the intersection of EF and CD be G. Compute GD.

25. Compute the number of digits in the decimal representation of

$$\prod_{n=1}^{2016} n^n.$$

Your score will be given by $\lfloor 25 \min\{(\frac{A}{C})^2, (\frac{C}{A})^2\}\rfloor$, where A is your answer and C is the actual answer.

26. 1000 unit cubes are stacked to form a cube with side length 10. A sphere of radius 10 is centered at the corner of the large cube. How many small cubes are cut by the sphere? In other words, how many small cubes have interior points which lie on the surface of the sphere? Your score will be given by $\lfloor 25 \min\{(\frac{A}{C})^2, (\frac{C}{A})^2\}\rfloor$, where A is your answer and C is the actual answer.

27. You play a game where you roll a fair 6-sided die repeatedly until the cumulative sum of your rolls is exactly a perfect cube, at which point the game ends. What is the expected number of rolls before the game ends? Your score will be given by $\lfloor 25 \min\{(\frac{A}{C})^2, (\frac{C}{A})^2\}\rfloor$, where A is your answer and C is the actual answer.