1. Carolina has a jar of red, green, and white jelly beans. She replaces 400 red beans with green ones, then 200 green beans with white ones. Initially, the percentage of red beans minus the percentage of green beans was $X\%$. After the replacements, this difference became $(X - 2.5)\%$. Compute the number of beans in the jar.

2. Six friends—Andrea, Blake, Camila, Dean, Ethan, and Francis—drink a combined total of 32 cups of boba. Each person drinks at least one cup, no two friends drink the same number of cups, and each cup is fully finished by a single person. One friend, known as the "Boba Champion", drinks as many cups as all the others combined. Compute the product of the number of cups of boba consumed by the five friends who aren't the Boba Champion.

3. The absolute values of all three roots of the polynomial $x^3 - 147x + c$ are primes. Compute $|c|$. (A prime number is a positive integer greater than 1 whose only positive divisors are 1 and itself.)

4. Compute the sum of the real roots of $(x^2 + 16x + 48)(x^2 + 24x + 128) = 44^{2^{2025}}$. (Roots are counted with multiplicity.)

5. Given real numbers s, m, and t such that $s^2 + m^2 + t^2 = 8$, $s^3 + m^3 + t^3 = 11$, and $s^4 + m^4 + t^4 = 25$, compute the value of $(s + m + t)(s + m - t)(s - m + t)(s - m - t)$.

6. Compute the number of integers between 1 and 2025 inclusive that *cannot* be represented as $x(\lceil x \rceil + \lfloor x \rfloor)$ for any positive real number x.

7. The Fibonacci numbers are defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all positive integers $n \geq 3$. Let $S = \{2^{F_2}, 2^{F_3}, ..., 2^{F_{27}}, 2^{F_{28}}\} = \{2, 4, ..., 2^{196418}, 2^{317811}\}$ be a set. If

$$P = \prod_{x \in S} \prod_{y \in S} x^{\log_3 y}$$

is the product of $x^{\log_3 y}$ across all ordered pairs of elements (x, y) in S, including when $x = y$, compute $\sqrt{\log_2 P}$.

8. Let Z be the set of all complex numbers z such that $|z| = 1$ and

$$z^8 + iz^6 - (1 + i)z^5 + (1 - i)z^3 + iz^2 - 1 = 0,$$

with the polar angle θ_z (the angle in $[0, 2\pi)$ that z makes with the positive real axis, measured counterclockwise) lying in the interval $[0, \pi)$. Compute the sum of all distinct values of θ_z for z in Z.

9. Define a sequence $\{a_n\}$ by $a_1 = 5$ and $a_{n+1} = \frac{5a_n + \sqrt{21a_n^2 + 4}}{2}$ for $n \geq 1$. Compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{a_n a_{n+1}}.$$

10. Let $f(x) = 4x + a$, $g(x) = 6x + b$, and $h(x) = 9x + c$. Let $S(a, b, c)$ be the set of the 3^{20} functions formed by all possible compositions of f, g, h a total of 20 times, and let $R(a, b, c)$ be the number of distinct roots over all functions in $S(a, b, c)$. Compute the *second smallest* possible value of $R(a, b, c)$ as a, b, c range over all reals.

TB. *This is an estimation question used for tiebreaking purposes. Ties on this test will be broken by absolute distance from the correct answer on this question.* Let

$$A = \sum_{k=0}^{5000^2-1} \frac{1}{k - 4999\left\lfloor \frac{k}{5000} \right\rfloor + 1}.$$

Estimate A in the decimal form $abcdef.ghij$ where $a, b, c, d, e, f, g, h, i, j$ are decimal digits each between 0 and 9, inclusive (leading zeros are possible).