1. Ethan likes to build sandcastles on the beach. Let t, h, e, b, a, c be integers such that $b + e \neq 0$ and

$$\frac{t^h e}{b + e} = a - ch.$$

If $b = 0, c = 1, t = 2$, and $h = 3$, compute a.

2. An isosceles triangle has legs of length $9! + 8!$, a base of length $2(9! - 8!)$, and a height of length $m \cdot 8!$ perpendicular to the base for some positive integer m. Compute m.

(Here, $n!$ represents the factorial of n, or the product of all positive integers up to n. For example, $3! = 3 \times 2 \times 1 = 6$ and $2! = 2 \times 1 = 2$.)

3. Parker writes down all of the five-digit positive integers that use each of the digits 1, 2, 3, 4, and 5 exactly once, and also satisfy these two properties:

- The 1 is next to the 3 and the 5.
- The 2 is to the left of the 4.

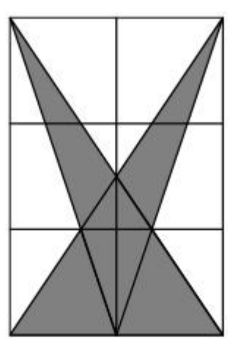
For example, one of the numbers that Parker writes down is 25134. Compute the sum of the smallest number and the greatest number that Parker writes down.

4. Carolina has a jar of red, green, and white jelly beans. She replaces 400 red beans with green ones, then 200 green beans with white ones. Initially, the percentage of red beans minus the percentage of green beans was $X\%$. After the replacements, this difference became $(X - 2.5)\%$. Compute the number of beans in the jar.

5. The boxes in the expression below are filled with the numbers 3, 4, 5, 6, and 7, so that each number is used exactly once, and the value of the expression is a prime number. Compute the value of the expression.

$$\square \times \square \times \square + \square \times \square$$

6. A 3×2 rectangle is divided into grid lines spaced one unit apart. The vertices on the grid are connected to form two overlapping triangles, both of which are shaded. Compute the area of the shaded region.

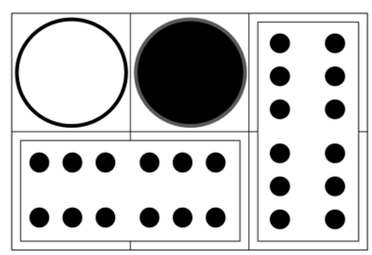


7. Six friends—Andrea, Blake, Camila, Dean, Ethan, and Francis—drink a combined total of 32 cups of boba. Each person drinks at least one cup, no two friends drink the same number of cups, and each cup is fully finished by a single person. One friend, known as the "Boba Champion", drinks as many cups as all the others combined. Compute the product of the number of cups of boba consumed by the five friends who aren't the Boba Champion.

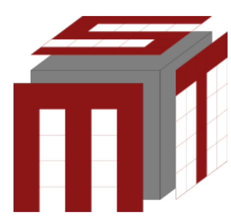
8. Cindy goes to sleep at midnight (12:00 A.M.) and wakes up after n hours, where n is an integer between 1 and 24, inclusive. Her sister, Sleepy Windy, goes to sleep at the same time and wakes up n^2 hours later. It turns out that their shared analog clock looks the same to both sisters when each of them wakes up. Compute the sum of all possible values of n. (An analog clock displays all times between 12:00 and 11:59 with each cycle repeated twice a day, but the sisters cannot tell apart the A.M. and P.M. by looking at the analog clock).

9. Professor Matrix Tessellation Laplace writes the following equation on a blank blackboard:
$$z = 0.$$
One by one, each of his 2025 students edits the equation by adding a new term:
$$x_1 y_1 + z = 0$$
$$x_2 y_2 + x_1 y_1 + z = 0$$
$$\vdots$$
When all 2025 students are done, compute the number of characters written on the blackboard. These include digits, variable names, operations, and the equal sign (there is always only one equation on the board). For example, $x_{12} y_{12} + z = 0$ has 10 characters.

10. Kevin has two indistinguishable 2×1 dominoes (labeled $6 - 6$), and Ryan has two 1×1 distinguishable checkers (one black and one white). They take turns placing one piece at a time on a fixed 2×3 grid, aligning with the gridlines without overlap. Each piece must be placed face up, preserving its markings. Compute the number of possible final configurations. One example is shown (rotated or reflected boards are considered distinct).



11. We have a $5 \times 5 \times 5$ cube made up of $1 \times 1 \times 1$ cardinal red and transparent pieces. When looking at the cube, we always see a cardinal red piece if it is behind any transparent pieces. When viewing each face straight-on, we can see the letters S, M, T on the top, front, and right side, respectively (as arranged in the diagram). Compute the maximum possible number of cardinal red pieces within this cube.
Note: cardinal red pieces are rendered black on your printed copy.



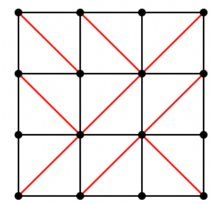
12. Compute the number of subsets A of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ which have the property that more than half of the elements in A are prime. For example, $A = \{2, 3, 4\}$ satisfies the condition. (A prime number is a positive integer greater than 1, whose only positive divisors are 1 and itself.)

13. Compute the number of triples of integers $-8 < a, b, c < 8$ that satisfy the following statements:

$$ab < 0,$$
$$ac < 0,$$
$$bc + a < 0.$$

14. Professor Matrix Tessellation Laplace rides his bike to his classroom everyday. To stop the Grand Theft Pedal in Stanford, he locks his bike using a 4-digit lock, and then shuffles his lock to a random combination. Each digit on the lock is a digit from 0 to 9 inclusive, and a flick is defined as changing a single digit by 1 (or between 0 and 9). Given that Professor Matrix Tessellation Laplace always finds the shortest combination of flicks, compute the average number of flicks he needs to unlock his bike.

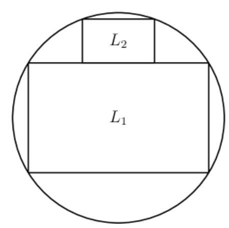
15. In a 3×3 grid, exactly one of the two possible diagonals is drawn in each of the nine cells. Compute the number of ways to choose the diagonals such that there is a continuous path along diagonals from the bottom-left vertex to the top-right vertex. One such grid is shown.



16. Let Ω be a circle. Draw two radii r_1 and r_2 of Ω that form a 40° angle, and let ω be a circle which is tangent to both of these radii and internally tangent to Ω. Let ω be tangent to Ω and r_1 at A and B, respectively, and let r_1 meet Ω at C. Compute $\angle ABC$.

17. Compute the sum of the real roots of $(x^2 + 16x + 48)(x^2 + 24x + 128) = 44^{2^{2025}}$. (Roots are counted with multiplicity.)

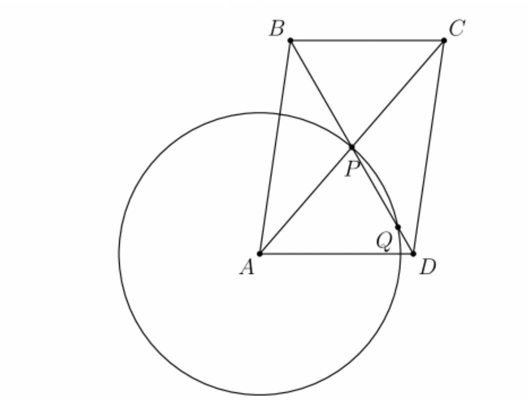
18. In the diagram shown, the rectangles L_1 and L_2 are similar, where corresponding sides of L_1 are $\frac{5}{2}$ times the length of the corresponding sides of L_2. Rectangle L_1 is inscribed in a circle ω, and rectangle L_2 meets ω at two points, with its long side lying on the long side of L_1. If r and s are the side lengths of L_1, where $s > r$, compute $\frac{s}{r}$.



19. An integer $m > 1$ is called *elementary* if its first digit is 1, its last digit is 1, and every other digit is 0. Let S be the set of the 2025 smallest elementary integers. Compute the greatest possible size of a subset T of S such that any two distinct elements in T are pairwise coprime.

20. Given real numbers s, m, and t such that $s^2 + m^2 + t^2 = 8$, $s^3 + m^3 + t^3 = 11$, and $s^4 + m^4 + t^4 = 25$, compute the value of $(s + m + t)(s + m - t)(s - m + t)(s - m - t)$.

21. Let $ABCD$ be a parallelogram with $AB > BC$ and diagonals intersecting at point P. Circle ω has its center at point A, radius AP, and intersects BD again at point Q on line segment PD. If $PQ = 3$, $QD = 1$, and $ABCD$ has integer side lengths, compute the length of AC.



22. Consider an axis-aligned 4 dimensional cube with side length 4, subdivided into 4^4 unit cubes. Cubey the penguin starts at a random unit cube. Each second, he can move to an adjacent unit cube along any one of the four axes. A fish is independently placed in a uniformly random unit cube (possibly the same one as Cubey's starting position) and the fish does not move (it's a fish out of water). Compute the expected time in seconds that Cubey will take to catch the fish, assuming he moves optimally to reach the fish as fast as possible.

23. Compute the number of integers between 1 and 2025 inclusive that *cannot* be represented as $x(\lceil x \rceil + \lfloor x \rfloor)$ for any positive real number x.
(Recall that $\lfloor x \rfloor$ is the greatest integer not exceeding x and $\lceil x \rceil$ is the smallest integer not less than x.)

24. Professor Matrix Tessellation Laplace arranges all of the integers from 1 to 99999 (inclusive) in the order that maximizes the gigantic number formed by concatenating them. In his order, the number 2025 appears as the i-th integer from left to right. Find i.

(Formally, he sets the permutation $\sigma = (\sigma_1, ..., \sigma_{99999})$ of $\{1, 2, ..., 99999\}$ such that the value of the concatenation $\overline{\sigma_1 \sigma_2 ... \sigma_{99999}}$ is maximized. Find the index i such that $\sigma_i = 2025$.)

25. Let $ABCD$ be a square centered at E with side length 10. Point P lies on the extension of line CD past point D. There exists a point Q on line AP such that lines EQ, BP, and AD are concurrent. If $EQ = 7\sqrt{2}$, compute $|AQ - DQ|$.

TB. *This is an estimation question used for tiebreaking purposes. Ties on this test will be broken by absolute distance from the correct answer to this question.*

Let s_0 be a square. One diagonal of s_0 is chosen, and a square s_1 is drawn with this diagonal as one of its sides. There are four possible such squares s_1; one is chosen out of these four uniformly at random. Choose s_2 in the same manner (so that it has a side which is a diagonal of s_1), and continue similarly with $s_3, s_4, ...$ Let p be the probability that square s_{20} shares an interior with s_0 (in other words, the area of their intersection is positive).
Estimate p in the decimal form $0.abcdefgh$, where a, b, c, d, e, f, g, h are decimal digits each between 0 and 9, inclusive.