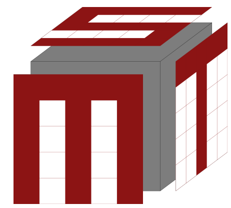
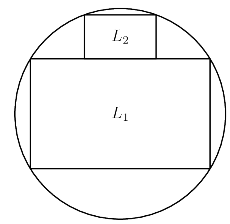
1. Let $\triangle ABC$ be an equilateral triangle with side length 6. Points D, E, and F lie on sides AB, BC, and CA respectively such that $\triangle DEF$ is equilateral and has area equal to $\triangle FAD$. Compute the side length of $\triangle DEF$.

2. We have a $5 \times 5 \times 5$ cube made up of $1 \times 1 \times 1$ cardinal red and transparent pieces. When looking at the cube, we always see a cardinal red piece if it is behind any transparent pieces. When viewing each face straight-on, we can see the letters S, M, T on the top, front, and right side, respectively (as arranged in the diagram). Compute the maximum possible number of cardinal red pieces within this cube.
Note: cardinal red pieces are rendered black on your printed copy.



3. Let Ω be a circle. Draw two radii r_1 and r_2 of Ω that form a 40° angle, and let ω be a circle which is tangent to both of these radii and internally tangent to Ω. Let ω be tangent to Ω and r_1 at A and B, respectively, and let r_1 meet Ω at C. Compute $\angle ABC$.

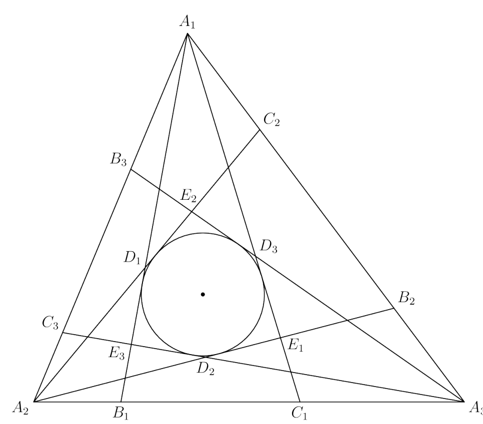
4. In the diagram shown, the rectangles L_1 and L_2 are similar, where corresponding sides of L_1 are $\frac{5}{2}$ times the length of the corresponding sides of L_2. Rectangle L_1 is inscribed in a circle ω, and rectangle L_2 meets ω at two points, with its long side lying on the long side of L_1. If r and s are the side lengths of L_1, where $s > r$, compute $\frac{s}{r}$.



5. Let α and β be two angles in $[0, 2\pi)$, and let $A = (\cos\alpha, \sin\alpha)$, $B = (\cos\beta, \sin\beta)$ and $C = (\cos\alpha + \cos\beta, \sin\alpha + \sin\beta)$. If one of the points has coordinates $\left(\frac{32}{25}, \frac{24}{25}\right)$, compute the area of $\triangle ABC$.

6. Let $\mathcal{P}$ be a parabola with its vertex at A, and let B and C be two other distinct points on $\mathcal{P}$. The focus F of $\mathcal{P}$ is the centroid of $\triangle ABC$. If $FB = 3$, compute the area of $\triangle ABC$.

7. Triangle $\triangle A_1A_2A_3$ has side lengths $A_1A_2 = 13$, $A_2A_3 = 14$, $A_3A_1 = 15$. Points B_i and C_i lie on $\overline{A_{i+1}A_{i+2}}$ such that $\angle B_iA_iC_i = 30^\circ$ for $1 \le i \le 3$, where $A_4 = A_1$ and $A_5 = A_2$. For $1 \le i \le 3$, let D_i be the intersection of $\overline{A_iB_i}$ and $\overline{A_{i+1}C_{i+1}}$ and E_i the intersection of $\overline{A_iC_i}$ and $\overline{A_{i+1}B_{i+1}}$, where $A_4 = A_1, B_4 = B_1, C_4 = C_1$. If $D_1E_3D_2E_1D_3E_2$ forms a convex hexagon with an inscribed circle of radius r, compute r.



8. Let $PABCD$ be a pyramid with rectangular base $ABCD$ and $\frac{\cos\angle APC}{\cos\angle BPD} = 2$. If $BP = 4$ and $DP = 9$, compute $AP + CP$.

9. Let $ABCD$ be a square centered at E with side length 10. Point P lies on the extension of line CD past point D. There exists a point Q on line AP such that lines EQ, BP, and AD are concurrent. If $EQ = 7\sqrt{2}$, compute $|AQ - DQ|$.

10. Let $AFDC$ be a rectangle. Construct points E and B outside of $AFDC$ such that $AB = BC = DE = EF = 45$ and $ABCDEF$ is a convex hexagon. Let $\mathcal{E}$ be an inscribed ellipse tangent to sides AB, BC, CD, DE, EF, FA at points U, V, W, X, Y, and Z, respectively. Points F_1 and F_2 with F_1 closer to B are the foci of $\mathcal{E}$ satisfying $\triangle ABC \cong \triangle F_1WF_2 \cong \triangle DEF$. Let Q be on line CD such that $F_1Y \perp ZQ$. Compute the area of quadrilateral F_1QYZ.

TB. *This is an estimation question used for tiebreaking purposes. Ties on this test will be broken by absolute distance from the correct answer to this question.*

The incircle of triangle ABC is tangent to AB at F such that $AF = 5$, $BF = 12$, and the area of the triangle is 20250.

Estimate $\cos \angle ABC$ in the decimal form $0.abcdefgh$ where a, b, c, d, e, f, g, h are decimal digits each between 0 and 9, inclusive.