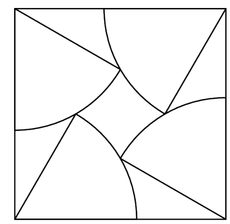
1. Using the numbers $3, 3, 8$, and 8 exactly once each, and only the operations of addition, subtraction, multiplication, and division, form an expression equal to 24. You may apply the operations in any order through the use of parentheses. Find and **submit this expression**.

2. The Stanford Tree buys 15 eggs to bake a cake. Unfortunately, n of these eggs are stale and one additional egg is rotten. The recipe requires the Tree to choose 3 eggs from the lot. A combination is called *bad* if it contains at least one stale egg and no rotten eggs and *awful* if it contains the rotten egg (even if no other eggs are stale). Let x denote the number of bad combinations and y denote the number of awful combinations. Given that the ratio of x to y is $40 : 13$, compute n.

3. Four slices of pizza are placed in a square box as shown in the diagram. Each pizza slice is a 60° sector of a circle with radius 1. Compute the side length of the box.



4. Compute the unique integer triple (a, b, c) satisfying $0 < a < b < c < 100$ and $a \cdot b + c = 200$ that minimizes $c - a$.

5. Alice has four sticks of lengths $3, 3, 5$, and x. She notices that it is possible to make a cyclic quadrilateral by joining these sticks at their endpoints, and no matter how she makes this cyclic quadrilateral, it always has a diagonal of length 6. Compute x.

6. Archimedes, Basil, Cleopatra, Diophantus, and Euclid each made exactly one claim, listed below in some order:

    1. At least two of the five statements are false.
    2. Each interior angle of a regular pentagon has angle measure 108°.
    3. 2025 is divisible by 4.
    4. Diophantus said the truth.
    5. Euclid said the truth.

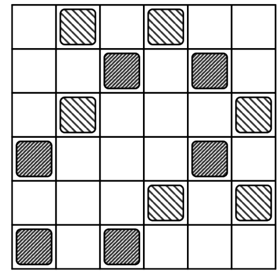
    Pythagoras is attempting to label each claim with the person who made it. A labeling is *determinable* if there exists a **unique** valid assignment of truth values (TRUE or FALSE) to the five claims such that the system of claims is consistent with the truth values. Compute the number of *determinable* labelings.
    For example, consider the labeling of Archimedes to 1, Basil to 2, Cleopatra to 3, Diophantus to 4, and Euclid to 5. Then one valid (but not unique) assignment of truth values is:

    $$1.\ \text{TRUE} \quad 2.\ \text{TRUE} \quad 3.\ \text{FALSE} \quad 4.\ \text{TRUE} \quad 5.\ \text{FALSE}.$$

7. Compute $\sin(\frac{\pi}{24})\sin(\frac{5\pi}{24}) + \sin(\frac{3\pi}{24})\sin(\frac{7\pi}{24}) + \sin(\frac{5\pi}{24})\sin(\frac{9\pi}{24}) + \cdots + \sin(\frac{19\pi}{24})\sin(\frac{23\pi}{24})$.

8. Compute the number of ordered pairs of positive integers (x, y) with $x + y \leq 20$ for which there exists a positive integer n satisfying $n = \operatorname{lcm}(n, x) - \gcd(n, y)$.

9. On a 6×6 grid, 12 identical pebbles are placed in the cells such that there are exactly 2 pebbles in every row and every column. Partition the pebbles into groups by declaring that two pebbles belong to the same group if one can reach the other by a sequence of moves between pebbles that lie in the same row or the same column. For example, in the grid shown, there are two groups.



   Given a valid arrangement A of these 12 pebbles, define $f(A)$ to be the number of groups in A. Let S be the set of all valid arrangements A. Compute $\sum_{A \in S} 2^{f(A)}$.

10. For positive integers p and q, there are exactly 14 ordered pairs of real numbers (a, b) with $0 < a, b < 1$ such that both $ap + 3b$ and $2a + bq$ are positive integers. Compute the number of possible ordered pairs (p, q).

11. Let ω_1 and ω_2 be circles such that ω_2 passes through the center of ω_1 and is also internally tangent to it. In addition, let AB be a chord of ω_1 tangent to ω_2 at some point T. Given that $AT = 9$ and $BT = 16$, compute the radius of ω_1.

12. For each positive integer n, define the coloring C_n as a coloring of all lattice points (i.e. points with integer coordinates) on the plane such that any two points $A = (A_x, A_y)$, $B = (B_x, B_y)$ have the same color if and only if $A_x + A_y$ and $B_x + B_y$ have the same remainder when divided by n. A positive integer n is called *broken* if in the coloring C_n there exist two points of different color and Euclidean distance a positive integer divisible by n. Find the sum of all broken positive integers less than or equal to 50.

13. There are thirteen lamps arranged in a circle in the complex plane, with lamp k (for $0 \leq k \leq 12$) located at $z_k = e^{2\pi i k/13}$. Lamp 0 is always on, while lamps 1 to 12 are initially turned off. For each $1 \leq r \leq 12$, toggling lamp r will flip it from on to off (and vice versa) and will also flip its neighbors, excluding lamp 0 if it is a neighbor. Let s be the sum of the positions z_k for lamps which are on. Compute the maximum possible value of $|s|$ over all configurations of switched on lamps reachable by flipping some subset of lamps from the initial state.

14. Let S be the set of all strings of length 15 formed from five 1s, 2s, and 3s. Say a string in S is *threnodic* if:

    - No two adjacent characters are the same, and
    - Through a sequence of removals of contiguous substrings 123, 231, and 312, the string can be deleted (note that the intermediate strings can have adjacent equal characters).

    Compute the number of threnodic strings in S.

15. Let $\triangle ABC$ be an acute triangle with circumcircle ω_1, and let D be a point on segment BC. Circle ω_2 is tangent to segment AD, segment BD, and ω_1. Circle ω_3 is tangent to segment AD, segment CD, and ω_1, and both circles are on the same side of BC as A. If circles ω_2 and ω_3 have radii 5 and 7, respectively, with centers 13 units apart, compute the sum of all possible lengths of the inradius of $\triangle ABC$.